Problem 6

Use equation (1.8) to find the fractions that are equivalent to the following repeating decimals:

$$0.61111\cdots$$

Solution

$$\begin{aligned}
0.61111\cdots &= 0.6 + 0.01 + 0.001 + \cdots \\
&= \frac{3}{5} + \frac{1}{100} + \frac{1}{1000} + \cdots \\
&= \frac{3}{5} + \sum_{i=0}^{\infty} \left(\frac{1}{100}\right) \frac{1}{10^i} \\
&= \frac{3}{5} + \sum_{i=0}^{\infty} \left(\frac{1}{100}\right) \left(\frac{1}{10}\right)^i \\
&= \frac{3}{5} + \frac{\frac{1}{100}}{1 - \left(\frac{1}{10}\right)} \\
&= \frac{11}{18}
\end{aligned}$$